Structure of Matter – part I – MidTest1
February 20, 2020

PROBLEM 1. The Gross Structure of One-Electron Systems [11 pnts]

Consider a 6f electron.

a) Calculate the polar angle of the angular momentum vector of a single 6f electron of which m=-2. [2 pnts]

b) Sketch the radial part of the 6f wave function (rR_{6f}). [2 pnts]

c) Consider all radial wave functions of hydrogen that have a similar shape as the 6f radial wave function. Which one of these is strongest bound and what is its binding energy. [3 pnt]

d) What is the binding energy in of a 6f electron in hydrogen-like Ar, i.e. Ar^{17+}(6f). [2 pnt]

e) What is the difference between the radial part of the 6f wave function of H and the one of Ar^{17+}, also indicate the reason for this difference [2 pnt]

PROBLEM 2. The Fine Structure of One-Electron Systems [13 pnts]

a) The fine structure levels are characterized by the quantum number j. What are the j values of a 5p electron. [1 pnt]

b) Briefly describe the physical mechanism that leads to the coupling of $\bar{l}$ and $\bar{s}$. [2 pnts]

c) Consider the hypothetical case that $l = 3$ and $s = 9/2$.

What are the possible values of j? [2 pnts]

d) *Back to the 5p electron*. Calculate the energy of the fine structure levels w.r.t. to the unperturbed 5p energy. The fine structure constant A=60 [cm^{-1}]. [1 pnt]

Hint: $V_{SO} = \frac{A}{2}(j(j+1) - l(l+1) - s(s+1))$.

e) The shift of the j levels is asymmetric w.r.t. the "unperturbed" 5d binding energy. Show that conservation of energy is not violated. [2 pnts]

f) Consider the upper, least bound j level. This system is put in an external magnetic field B, sketch the behavior of the binding energies of the relevant m_j states as a function of B. [2 pnt]

g) What does happen if the external magnetic field becomes much stronger than the internal magnetic field. [1 pnt]

h) Estimate the order of magnitude of the external magnetic field for which the fine structure of the atom breaks down. [2 pnt]

Hint: $g_j = 1 + \frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)}$ and $\mu_B = 0.47$ [cm^{-1}/T]